2021 추계공동학술대회

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Stochastic Programming Approach for the E-commerce Supply Chain Network Design with On-demand Warehousing System

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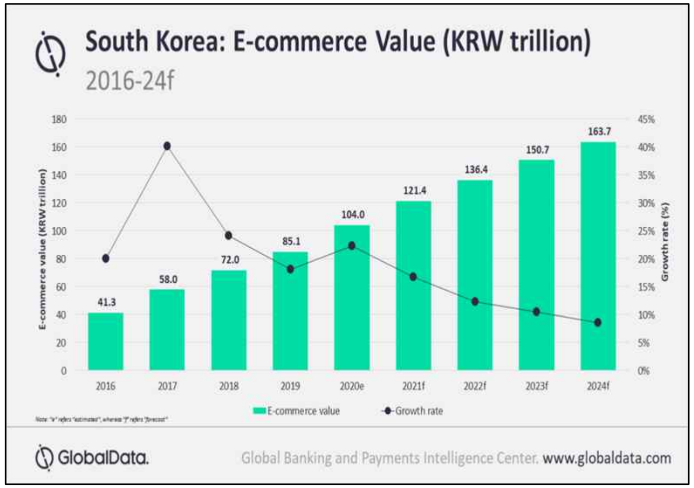
Outline

I. Introduction

Growth of E-commerce



Source: GlobalData (2020)

Online shopping hits record high in Q2 amid pandemic

Source: The Korea Herald (2021)

Number of new e-commerce retailers are increased by 145% in Coupang

Source: Consumer News (2020)

E-commerce retailers

IoT Logistics Covid-19

E-commerce retailers' properties

E-commerce retailers (Retailers)

1. Low-capital business

2. Warehouse with small space

Traditional solutions

1. Build warehouse infrastructure
   → High setup cost

2. Lease warehouse from traditional warehouse operators (Long-term)
   → Low flexibility

On-demand warehousing

Retailers
Need Services

On-Demand Warehousing
Make Connections

Warehouse Providers
Provide Services

Source: FLEXE.com

FLEXE

마이창고
www.mychango.com

How is on-demand warehousing different?

Low-risk way to test new strategies and keep up with rising customer expectations

- Connect warehouse providers who have **excess capacity** and retailers who need **flexible solutions**
- Secure warehousing and fulfillment **solutions quickly**
- Create a **distribution-network strategy** that's as dynamic as retailer's business
- Match **varying demand** and manage the unexpected throughout the year

Research questions

Decision Maker: E-commerce retailer

RQ1: Advantages of on-demand warehousing system

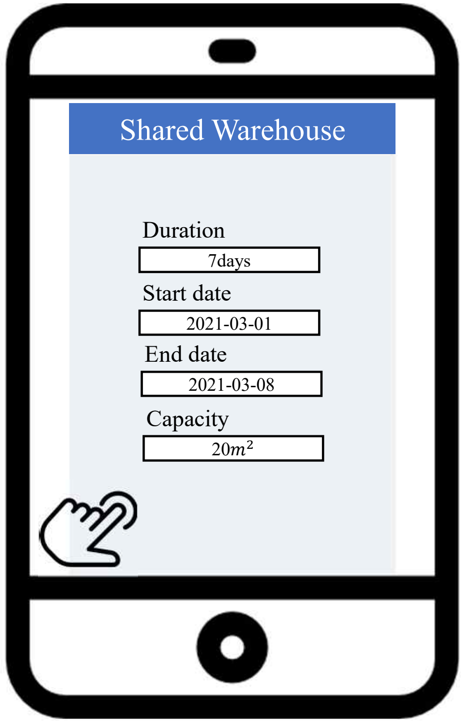
- Total cost
- Utilization of warehouses

RQ2: Supply chain design

- Inherent uncertainty
- Effective solution approach

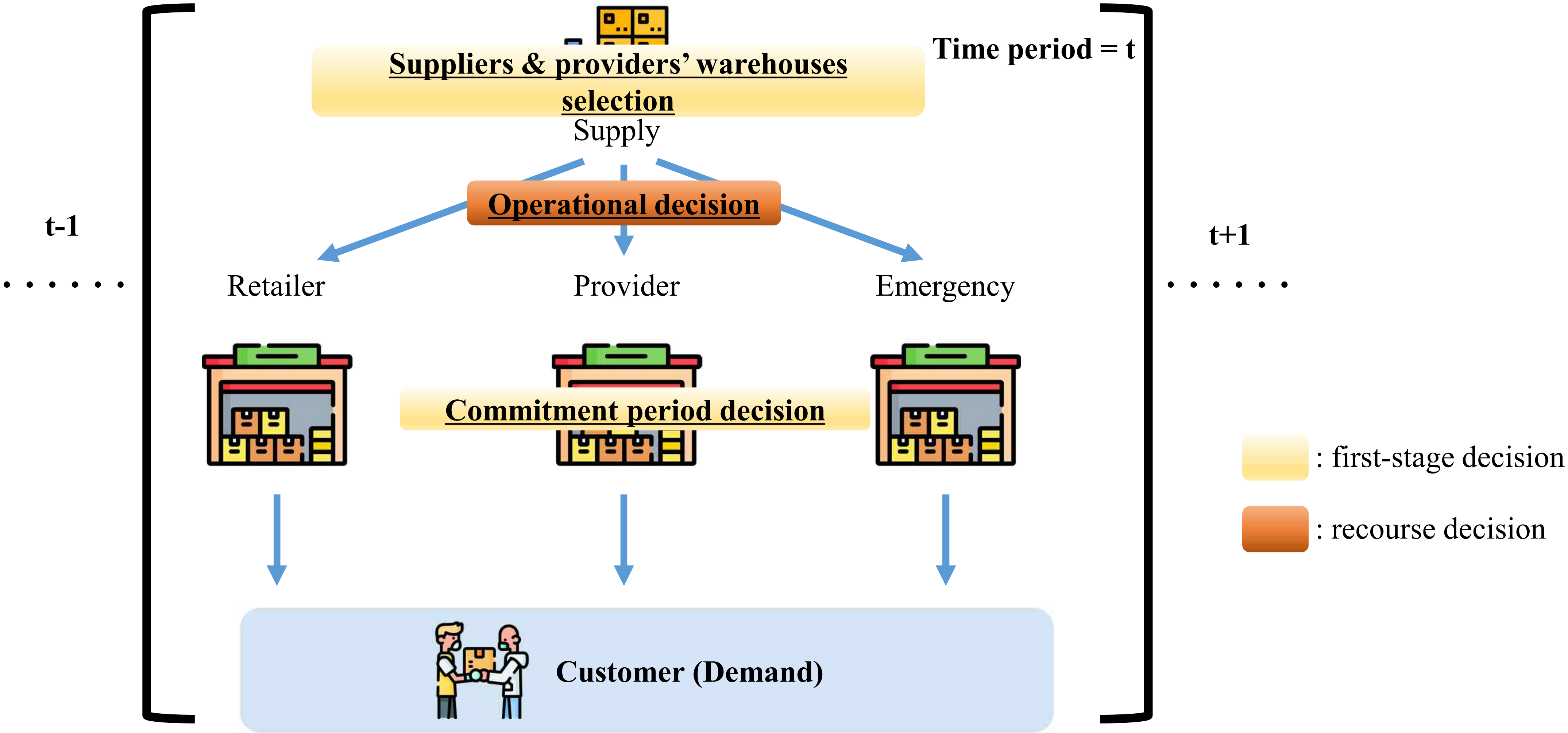
RQ3: Warehouse commitments

- Varying commitment cost
- Available shared warehouses



II. Problem Description and Mathematical Model

Problem description



Problem description

First-stage decision

Suppliers selection & Commitment periods decision

- Suppliers and providers' warehouses selection
- Warehouse (Retailer, Provider, Emergency)

How long do we use the provider's warehouse?

$m = |\mathcal{M}|$ (Long term) $m = 1$ (Short term)

Cheap ⟷ **Expensive**

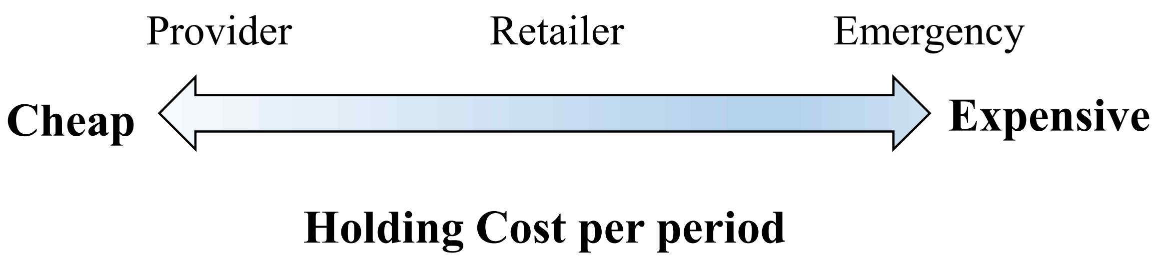
commitment cost: $m\ \alpha\gamma^m$

α :commitment cost for a day
m :the period of commitment
γ :discount factor

Problem description

Recourse decision
Operational decision

- Inventory holding decision in warehouse (Retailer, Provider, Emergency)
- Transportation between warehouses, suppliers, and customers
- Stockout



Assumptions

- Stockout → lost sales
- No lead time
- Demand
- Supply
- Capacity of warehouses
- Maximum period of commitment

Uncertain

Given

Nomenclature

Indices and sets

$\mathcal{T}$	set of periods, $t \in \mathcal{T} = \{1, 2, \cdots, T\}$
$\mathcal{I}$	set of items, $i \in \mathcal{I} = \{1, 2, \cdots, I\}$
$\mathcal{J}$	set of suppliers, $j \in \mathcal{J} = \{1, 2, \cdots, J\}$
$\mathcal{K}$	set of provider's warehouses, $k \in \mathcal{K} = \{1, 2, \cdots, K\}$
$\mathcal{M}$	set of available commitment periods, $m \in \mathcal{M} = \{1, 2, \cdots, M\}$
Ω	set of scenarios, $\omega \in \Omega$

Parameters

Uncertainty

D_{it}^{ω}	demand of items i at period t under scenario ω
S_{ijt}^{ω}	supply of items i from supplier j at period t under scenario ω
C^r	capacity of retailer's warehouse
C^k	capacity of provider's warehouse k
F_j	investment cost to utilize items of supplier j
h_i^r	inventory holding cost of retailer's warehouse per unit per period for item i
h_i^k	inventory holding cost of provider's warehouse k per unit per period for item i
h_i^e	inventory holding cost of emergency warehouse per unit per period for item i
α	commitment cost for a day
b_i	delivery service cost of a logistics company for a unit of item i
c_{ij}^r	transportation cost for a unit of item i from supplier j to retailer's warehouse
c_{ij}^k	transportation cost for a unit of item i from supplier j to provider's warehouse k
c_{ij}^e	transportation cost for a unit of item i from supplier j to emergency warehouse
γ	discount count factor of commitment cost
p_ω	probability that scenario ω occurred

Nomenclature

Decision variables

g_{mt}^{k}	1 if a m period commitment is made at period t for provider's warehouse k, 0 otherwise
r_{mt}^{k}	1 if items can be stored at provider's warehouse k due to the m period commitment at period t, 0 otherwise
y_j	1 if items of supplier j is available entire time horizon, 0 otherwise

First-stage decision

$v_{it}^{r\omega}$	number of item i held in inventory at retailer's warehouse from period t to $t+1$ under scenario ω
$v_{it}^{k\omega}$	number of item i held in inventory at provider's warehouse of k from period t to $t+1$ under scenario ω
$v_{it}^{e\omega}$	number of item i held in inventory at emergency warehouse from period t to $t+1$ under scenario ω
$x_{ijt}^{r\omega}$	number of item i transported from supplier j to retailer's warehouse at period t under scenario ω
$x_{ijt}^{k\omega}$	number of item i transported from suppliers j to provider's warehouse k at period t under scenario ω
$x_{ijt}^{e\omega}$	number of item i transported from suppliers j to emergency warehouse at period t under scenario ω
$u_{it}^{r\omega}$	number of item i delivered to customers from retailer's warehouse at period t under scenario ω by a logistics company
$u_{it}^{k\omega}$	number of item i delivered to customers from provider's warehouse k at period t under scenario ω by a logistics company
$u_{it}^{e\omega}$	number of item i delivered to customers from emergency warehouse at period t under scenario ω by a logistics company
z_{it}^{ω}	lost sales of item i at period t under scenario ω

Recourse decision

Two-stage stochastic programming model

First stage problem

$$\min \underbrace{\sum_{j \in \mathcal{J}} F_j y_j}_{} + \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} m \alpha \gamma^m g_{mt}^k + \sum_{\omega \in \Omega} p_\omega Q(y, r, \omega) \quad (1)$$

$$\text{s.t.} \sum_{t}^{\min\{t+m-1, |\mathcal{T}|\}} g_{mt}^k \leq 1, \quad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T}, \quad (2)$$

$$\sum_{\tau=\max\{t-m+1,1\}}^{t} g_{m\tau}^k = r_{mt}^k, \quad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T}, \quad (3)$$

$$\sum_{m \in \mathcal{M}} r_{mt}^k \leq 1, \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \quad (4)$$

$$\sum_{m \in \mathcal{M}} g_{mt}^k \leq 1, \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \quad (5)$$

$$r_{mt}^k, g_{mt}^k \in \{0, 1\}, \quad \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T}, \quad (6)$$

$$y_j \in \{0, 1\}, \quad \forall j \in \mathcal{J}. \quad (7)$$

Commitments (2)–(6)

Supply (7)

Investment cost for suppliers

Commitment cost for provider's warehouses

Optimal value of the second stage recourse problem

Two-stage stochastic programming model

Second stage recourse problem

$$Q(y,r,\omega) = \min \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} \Bigg(h_i^r v_{it}^{r\omega} + h_i^e v_{it}^{e\omega} + \sum_{k\in\mathcal{K}} h_i^k v_{it}^{k\omega} + b_i \Big(u_{it}^{r\omega} + u_{it}^{e\omega} + \sum_{k\in\mathcal{K}} u_{it}^{k\omega} \Big) + \beta_i z_{it}^{\omega} + \sum_{j\in\mathcal{J}} \Big(c_{ij}^r x_{ijt}^{r\omega} + c_{ij}^e x_{ijt}^{e\omega} + \sum_{k\in\mathcal{K}} c_{ij}^k x_{ijt}^{k\omega} \Big) \Bigg) \tag{8}$$

$$\text{s.t.}\quad x_{ijt}^{r\omega} + \sum_{k\in\mathcal{K}} x_{ijt}^{k\omega} + x_{ijt}^{e\omega} \le S_{ijt}^{\omega} y_j, \qquad \forall i\in\mathcal{I}, \forall j\in\mathcal{J}, \forall t\in\mathcal{T}, \tag{9}$$

$$u_{it}^{r\omega} + v_{it}^{r\omega} = v_{it-1}^{r\omega} + \sum_{j\in\mathcal{J}} x_{ijt}^{r\omega}, \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \tag{10}$$

$$u_{it}^{k\omega} + v_{it}^{k\omega} = v_{it-1}^{k\omega} + \sum x_{ijt}^{k\omega}, \qquad \forall i\in\mathcal{I}, \forall k\in\mathcal{K}, \forall t\in\mathcal{T}, \tag{11}$$

$$u_{it}^{e\omega} + v_{it}^{e\omega} = v_{it-1}^{e\omega} + \sum_{j\in\mathcal{J}} x_{ijt}^{e\omega}, \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \tag{12}$$

$$u_{it}^{r\omega} + \sum_{k\in\mathcal{K}} u_{it}^{k\omega} + u_{it}^{e\omega} + z_{it}^{\omega} \ge D_{it}^{\omega}, \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \tag{13}$$

$$\sum_{i\in\mathcal{I}} v_{it}^{r\omega} \le C^r, \qquad \forall t\in\mathcal{T}, \tag{14}$$

$$\sum_{i\in\mathcal{I}} v_{it}^{k\omega} \le C^k \sum_{m=1}^{M} r_{mt}^k, \qquad \forall k\in\mathcal{K}, \forall t\in\mathcal{T}, \tag{15}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} x_{jt}^{r\omega} \le C^r, \qquad \forall t\in\mathcal{T}, \tag{16}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} x_{jt}^{k\omega} \le C^k \sum_{m\in\mathcal{M}} r_{mt}^k, \qquad \forall k\in\mathcal{K}, \forall t\in\mathcal{T}, \tag{17}$$

$$v_{i0}^{r\omega}, v_{i0}^{k\omega}, v_{i0}^{e\omega} = 0, \qquad \forall i\in\mathcal{I}, \forall k\in\mathcal{K}, \tag{18}$$

$$x_{ijt}^{r\omega}, x_{ijt}^{k\omega}, x_{ijt}^{e\omega} \ge 0, \qquad \forall i\in\mathcal{I}, \forall j\in\mathcal{J}, \forall k\in\mathcal{K}, \forall t\in\mathcal{T}, \tag{19}$$

$$u_{it}^{r\omega}, u_{it}^{k\omega}, u_{it}^{e\omega}, v_{it}^{r\omega}, v_{it}^{k\omega}, v_{it}^{e\omega} \ge 0, \qquad \forall i\in\mathcal{I}, \forall k\in\mathcal{K}, \forall t\in\mathcal{T}, \tag{20}$$

$$z_{it}^{\omega} \ge 0, \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}. \tag{21}$$

- Inventory holding cost
- Delivery service cost for a logistics company
- Stockout cost
- Transportation cost

(9) - (13) : Distribution constraints

(14) - (17) : Capacity constraints

Two-stage stochastic programming model

Compact representation

First stage problem:

$$\min \quad \mathbf{f}^\intercal \mathbf{y} + \mathbf{e}^\intercal \mathbf{g} + \sum_{\omega \in \Omega} p_\omega Q(\mathbf{y}, \mathbf{r}, \omega) \tag{22}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{g} \leq \mathbf{1}, \tag{23}$$

$$\mathbf{B}\mathbf{g} = \mathbf{r}, \tag{24}$$

$$\mathbf{G}\mathbf{r} \leq \mathbf{1}, \tag{25}$$

$$\mathbf{y} \in \{0,1\}^{|\mathcal{J}|}, \tag{26}$$

$$\mathbf{g}, \mathbf{r} \in \{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{27}$$

Second stage recourse problem:

$$Q(\mathbf{y}, \mathbf{r}, \omega) =$$

$$\min \quad \mathbf{h}^\intercal \mathbf{v}_\omega + \mathbf{b}^\intercal \mathbf{u}_\omega + \boldsymbol{\beta}^\intercal \mathbf{z}_\omega + \mathbf{c}^\intercal \mathbf{x}_\omega \tag{28}$$

$$\text{s.t.} \quad \mathbf{P}\mathbf{x}_\omega \leq \mathbf{S}_\omega \bar{\mathbf{y}}, \tag{29}$$

$$\mathbf{U}\mathbf{u}_\omega + \mathbf{V}\mathbf{v}_\omega - \mathbf{T}\mathbf{x}_\omega = \mathbf{0}, \tag{30}$$

$$\mathbf{K}\mathbf{u}_\omega + \mathbf{R}\mathbf{z}_\omega \geq \mathbf{D}_\omega, \tag{31}$$

$$\mathbf{M}\mathbf{v}_\omega^r \leq \mathbf{C}^r, \tag{32}$$

$$\mathbf{H}\mathbf{v}_\omega^k \leq \mathbf{C}^k \bar{\mathbf{r}} \tag{33}$$

$$\mathbf{E}\mathbf{x}_\omega^r \leq \mathbf{C}^r, \tag{34}$$

$$\mathbf{L}\mathbf{x}_\omega^k \leq \mathbf{C}^k \bar{r}, \tag{35}$$

$$\mathbf{v}_\omega, \mathbf{u}_\omega, \mathbf{z}_\omega, \mathbf{x}_\omega \geq \mathbf{0}. \tag{36}$$

$$\mathbf{x}_\omega{}^\intercal := [\ (\mathbf{x}_\omega^r)^\intercal,\ (\mathbf{x}_\omega^k)^\intercal,\ (\mathbf{x}_\omega^e)^\intercal\ ], \qquad \mathbf{v}_\omega{}^\intercal := [\ (\mathbf{v}_\omega^r)^\intercal,\ (\mathbf{v}_\omega^k)^\intercal,\ (\mathbf{v}_\omega^e)^\intercal\ ]$$

III. Solution Approach

Sample average approximation

True problem

$$\begin{aligned}\min \quad & \mathbf{f}^\top \mathbf{y} + \mathbf{e}^\top \mathbf{g} + \sum_{\omega \in \Omega} p_\omega Q(\mathbf{y}, \mathbf{r}, \omega) \\ \text{s.t.} \quad & (23) - (27).\end{aligned}$$

Sample Average Approximation (SAA) problem

$$\begin{aligned}\min \quad & \mathbf{f}^\top \mathbf{y} + \mathbf{e}^\top \mathbf{g} + \frac{1}{N} \sum_{n=1}^{N} Q(\mathbf{y}, \mathbf{r}, \omega_n) \\ \text{s.t.} \quad & (23) - (27).\end{aligned}$$

- Sampling methodology → Obtain estimates of **upper and lower bound** on the optimal value of the *True problem*

- $(\omega_1, \cdots, \omega_N)$ → Generate a sample of scenarios (N independent scenarios) → Monte Carlo sampling

- *SAA problem* $\xrightarrow{\text{Approximation}}$ *True problem* (Optimal objective value: ψ^*)

Lower bound estimation

$$\min_{\mathbf{y}\in Y,\mathbf{g}\in G,\mathbf{r}\in R} \left\{ \mathbf{f}^\intercal \mathbf{y} + \mathbf{e}^\intercal \mathbf{g} + \frac{1}{N}\sum_{n=1}^{N} Q(\mathbf{y},\mathbf{r},\omega_n^m) \right\}$$

[SAA problem (sample *m*)]
$m = 1, \cdots, M$

- $(\omega_1^m, \cdots, \omega_N^m)$ → Generate *M* independent sample sets of scenarios
  → Monte Carlo sampling
- $\hat{\psi}_N^m$ → Optimal objective value
- $\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m, \hat{\mathbf{r}}_N^m$ → Optimal solution

Lower bound estimation

- Lower bound

$$\bar{\psi}_{MN} := \frac{1}{M}\sum_{m=1}^{M}\hat{\psi}_N^m \leq \psi^*$$

- Variance of $\bar{\psi}_{MN}$

$$\sigma^2_{\bar{\psi}_{MN}} := \frac{1}{M(M-1)}\sum_{m=1}^{M}\left(\hat{\psi}_N^m - \bar{\psi}_{MN}\right)^2$$ (Derived from *Central Limit Theorem*)

- $m^* = \mathrm{argmin}_{m\in\{1,\cdots,M\}}\hat{\psi}_N^m$

$$\hat{\mathbf{y}} := \hat{\mathbf{y}}_N^{m^*},\ \hat{\mathbf{g}} := \hat{\mathbf{g}}_N^{m^*},\ \hat{\mathbf{r}} := \hat{\mathbf{r}}_N^{m^*}$$

Upper bound estimation

- Determine the solution for upper bound

for each $m \in \{1, \cdots, M\}$ **do**

$\bar{f}_{N'}(\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m, \hat{\mathbf{r}}_N^m) \leftarrow \mathbf{f}^\intercal \hat{\mathbf{y}}_N^m + \mathbf{e}^\intercal \hat{\mathbf{g}}_N^m + \frac{1}{N'} \sum_{n=1}^{N'} Q(\hat{\mathbf{y}}_N^m, \hat{\mathbf{r}}_N^m, \omega_n)$

end for

$$m^* \leftarrow \operatorname{argmin}_m \ \bar{f}_{N'}(\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m, \hat{\mathbf{r}}_N^m), \ \forall m \in \{1, \cdots, M\}$$

$$\hat{\mathbf{y}} := \hat{\mathbf{y}}_N^{m^*}, \ \hat{\mathbf{g}} := \hat{\mathbf{g}}_N^{m^*}, \ \hat{\mathbf{r}} := \hat{\mathbf{r}}_N^{m^*}$$

- Upper bound

$$\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) := \mathbf{f}^\intercal \hat{\mathbf{y}} + \mathbf{e}^\intercal \hat{\mathbf{g}} + \frac{1}{N'} \sum_{n=1}^{N'} Q(\hat{\mathbf{y}}, \hat{\mathbf{r}}, \omega_n), \quad \psi^* \leq \bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}), \ N \ll N'$$

- Variance of $\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$

$$\sigma_{N'}^2(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) := \frac{1}{N'(N'-1)} \sum_{n=1}^{N'} \left(\mathbf{f}^\intercal \hat{\mathbf{y}} + \mathbf{e}^\intercal \hat{\mathbf{g}} + Q(\hat{\mathbf{y}}, \hat{\mathbf{r}}, \omega_n) - \hat{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) \right)^2$$

Optimality gap estimation

See Appendix A.

- Optimality gap of the solution $\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}$

$$Gap_{MNN'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) := \hat{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) - \hat{\psi}_{MN}$$

$$Gap^{rel}_{MNN'} \leftarrow \frac{(\hat{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) - \bar{\psi}_{MN})}{\bar{\psi}_{MN}} \times 100(\%)$$

- Variance of $Gap_{MNN'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$

$$\sigma^2_{Gap_{MNN'}} := \sigma^2_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) + \sigma^2_{\hat{\psi}_{MN}}$$

- If) $Gap^{rel}_{MNN'} < \epsilon_{saa}$

  Otherwise) $N \leftarrow N + 20$, and repeat the process

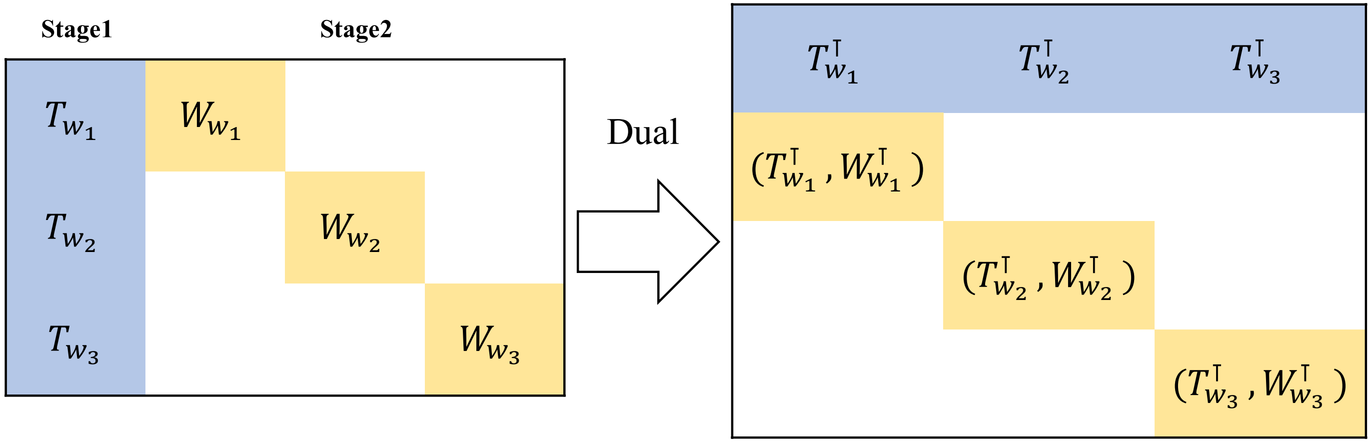
Why use Benders decomposition?

- $$\min_{\mathbf{y}\in Y,\mathbf{g}\in G,\mathbf{r}\in R}\left\{\mathbf{f}^\intercal\mathbf{y}+\mathbf{e}^\intercal\mathbf{g}+\frac{1}{N}\sum_{n=1}^{N}Q(\mathbf{y},\mathbf{r},\omega_n^m)\right\} \Longrightarrow$$ Spend a lot of time to compute with small size N

- ✓ With many realizations of scenarios, the two-stage stochastic problems become large

  ✓ Utilize the special structure (block diagonal structure) of stochastic programs

  ⇒ **<u>Benders decomposition</u>**

- 

Benders decomposition

Full master problem

Stage 1

$$\min \quad \mathbf{f}^\intercal \mathbf{y} + \mathbf{e}^\intercal \mathbf{g} + \sum_{\omega \in \Omega} p_\omega Q(\mathbf{y}, \mathbf{r}, \omega) \tag{22}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{g} \leq \mathbf{1}, \tag{23}$$

$$\mathbf{B}\mathbf{g} = \mathbf{r}, \tag{24}$$

$$\mathbf{G}\mathbf{r} \leq \mathbf{1}, \tag{25}$$

$$\mathbf{y} \in \{0,1\}^{|\mathcal{J}|}, \tag{26}$$

$$\mathbf{g}, \mathbf{r} \in \{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{27}$$

Stage 2

$Q(\mathbf{y}, \mathbf{r}, \omega) =$

$$\min \quad \mathbf{h}^\intercal \mathbf{v}_\omega + \mathbf{b}^\intercal \mathbf{u}_\omega + \boldsymbol{\beta}^\intercal \mathbf{z}_\omega + \mathbf{c}^\intercal \mathbf{x}_\omega \tag{28}$$

$$\text{s.t.} \quad \mathbf{P}\mathbf{x}_\omega \leq \mathbf{S}_\omega \mathbf{y}, \qquad (\boldsymbol{\pi}_\omega) \tag{29}$$

$$\mathbf{U}\mathbf{u}_\omega + \mathbf{V}\mathbf{v}_\omega - \mathbf{T}\mathbf{x}_\omega = \mathbf{0}, \qquad (\boldsymbol{\mu}_\omega) \tag{30}$$

$$\mathbf{K}\mathbf{u}_\omega + \mathbf{R}\mathbf{z}_\omega \geq \mathbf{D}_\omega, \qquad (\boldsymbol{\nu}_\omega) \tag{31}$$

$$\mathbf{M}\mathbf{v}_\omega^r \leq \mathbf{C}^r, \qquad (\boldsymbol{\lambda}_\omega) \tag{32}$$

$$\mathbf{H}\mathbf{v}_\omega^k \leq \mathbf{C}^k \mathbf{r}, \qquad (\boldsymbol{\rho}_\omega) \tag{33}$$

$$\mathbf{E}\mathbf{x}_\omega^r \leq \mathbf{C}^r, \qquad (\boldsymbol{\delta}_\omega) \tag{34}$$

$$\mathbf{L}\mathbf{x}_\omega^k \leq \mathbf{C}^k \mathbf{r}, \qquad (\boldsymbol{\kappa}_\omega) \tag{35}$$

$$\mathbf{v}_\omega, \mathbf{u}_\omega, \mathbf{z}_\omega, \mathbf{x}_\omega \geq \mathbf{0}. \tag{36}$$

Reformulation

$$\min \quad \mathbf{f}^\intercal \mathbf{y} + \mathbf{e}^\intercal \mathbf{g} + \sum_{\omega \in \Omega} p_\omega \theta_\omega \tag{37}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{g} \leq \mathbf{1}, \tag{38}$$

$$\mathbf{B}\mathbf{g} = \mathbf{r}, \tag{39}$$

$$\mathbf{G}\mathbf{r} \leq \mathbf{1}, \tag{40}$$

$$\theta_\omega \geq (\mathbf{a}_\omega^i)^\intercal \mathbf{y} + (\mathbf{c}_\omega^i)^\intercal \mathbf{r} + d_\omega^i, \qquad \forall i \in XP(\boldsymbol{\Pi}_\omega), \forall \omega \in \Omega, \tag{41}$$

$$\mathbf{y} \in \{0,1\}^{|\mathcal{J}|}, \tag{42}$$

$$\mathbf{g}, \mathbf{r} \in \{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{43}$$

where

$\mathbf{a}_\omega{}^\intercal := \boldsymbol{\pi}_\omega{}^\intercal \mathbf{S}_\omega$

$\mathbf{c}_\omega{}^\intercal := (\boldsymbol{\rho}_\omega + \boldsymbol{\kappa}_\omega)^\intercal \mathbf{C}^k$

$d_\omega := \boldsymbol{\nu}_\omega{}^\intercal \mathbf{D}_\omega + (\boldsymbol{\lambda}_\omega + \boldsymbol{\delta}_\omega)^\intercal \mathbf{C}^r$

$\boldsymbol{\Pi}_\omega :=$ feasible region of dual variables $(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\lambda}, \boldsymbol{\rho}, \boldsymbol{\delta}, \boldsymbol{\kappa})$ under scenario ω

$XP(\boldsymbol{\Pi}_\omega) :=$ finite set of extreme points of $\boldsymbol{\Pi}_\omega$

Benders decomposition

Full master problem (reformulation)

$$\min \quad \mathbf{f}^\intercal \mathbf{y} + \mathbf{e}^\intercal \mathbf{g} + \sum_{\omega \in \Omega} p_\omega \theta_\omega \tag{37}$$

$$\text{s.t.} \quad \mathbf{Ag} \le \mathbf{1}, \tag{38}$$

$$\mathbf{Bg} = \mathbf{r}, \tag{39}$$

$$\mathbf{Gr} \le \mathbf{1}, \tag{40}$$

$$\theta_\omega \ge (\mathbf{a}_\omega^i)^\intercal \mathbf{y} + (\mathbf{c}_\omega^i)^\intercal \mathbf{r} + d_\omega^i, \qquad \forall i \in XP(\mathbf{\Pi}_\omega), \forall \omega \in \Omega, \tag{41}$$

$$\mathbf{y} \in \{0,1\}^{|\mathcal{J}|}, \tag{42}$$

$$\mathbf{g}, \mathbf{r} \in \{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{43}$$

where

$\mathbf{a}_\omega{}^\intercal := \boldsymbol{\pi}_\omega{}^\intercal \mathbf{S}_\omega$

$\mathbf{c}_\omega{}^\intercal := (\boldsymbol{\rho}_\omega + \boldsymbol{\kappa}_\omega)^\intercal \mathbf{C}^k$

$d_\omega := \boldsymbol{\nu}_\omega{}^\intercal \mathbf{D}_\omega + (\boldsymbol{\lambda}_\omega + \boldsymbol{\delta}_\omega)^\intercal \mathbf{C}^r$

$\mathbf{\Pi}_\omega :=$ feasible region of dual variables $(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\lambda}, \boldsymbol{\rho}, \boldsymbol{\delta}, \boldsymbol{\kappa})$ under scenario ω

$XP(\mathbf{\Pi}_\omega) :=$ finite set of extreme points of $\mathbf{\Pi}_\omega$

- Number of variables has been reduced substantially
- Number of constraints can be extremely large due to the large size of extreme points
  - Overcome using cutting plane method (i.e., Benders decomposition)
  - Involve only subsets of Constraint (41)

Benders decomposition

Master problem (MP)

$$\min \quad \mathbf{f}^\intercal \mathbf{y} + \mathbf{e}^\intercal \mathbf{g} + \sum_{\omega \in \Omega} p_\omega \theta_\omega \tag{44}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{g} \le \mathbf{1}, \tag{45}$$

$$\mathbf{B}\mathbf{g} = \mathbf{r}, \tag{46}$$

$$\mathbf{G}\mathbf{r} \le \mathbf{1}, \tag{47}$$

$$\theta_\omega \ge (\mathbf{a}_\omega^t)^\intercal \mathbf{y} + (\mathbf{c}_\omega^t)^\intercal \mathbf{r} + d_\omega^t, \qquad \forall t \in \{1, \cdots, n\}, \forall \omega \in \Omega, \tag{48}$$

$$\mathbf{y} \in \{0,1\}^{|\mathcal{J}|}, \tag{49}$$

$$\mathbf{g}, \mathbf{r} \in \{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{50}$$

$Optimal\ solutions \rightarrow \bar{\mathbf{y}}, \bar{\mathbf{g}}, \bar{\mathbf{r}}, \bar{\theta}_\omega$

$\underset{(Z_{LB})}{Lower\ bound} \leftarrow \mathbf{f}^\intercal \bar{\mathbf{y}} + \mathbf{e}^\intercal \bar{\mathbf{g}} + \sum_{\omega \in \Omega} p_\omega \bar{\theta}_\omega$

$\bar{\mathbf{y}}, \bar{\mathbf{g}}, \bar{\mathbf{r}}, \bar{\theta}_\omega$

$until\ \ (Z_{UB} - Z_{LB} < \epsilon \times Z_{LB})$

Optimality cut

Sub problem for scenario ω (SUB(ω))

$For\ \omega = 1, \cdots, |\Omega|$

$Q(\bar{\mathbf{y}}, \bar{\mathbf{r}}, \omega) =$

$$\min \quad \mathbf{h}^\intercal \mathbf{v}_\omega + \mathbf{b}^\intercal \mathbf{u}_\omega + \boldsymbol{\beta}^\intercal \mathbf{z}_\omega + \mathbf{c}^\intercal \mathbf{x}_\omega \tag{51}$$

$$\text{s.t.} \quad \mathbf{P}\mathbf{x}_\omega \le \mathbf{S}_\omega \bar{\mathbf{y}}, \qquad (\boldsymbol{\pi}_\omega), \tag{52}$$

$$\mathbf{U}\mathbf{u}_\omega + \mathbf{V}\mathbf{v}_\omega - \mathbf{T}\mathbf{x}_\omega = \mathbf{0}, \qquad (\boldsymbol{\mu}_\omega), \tag{53}$$

$$\mathbf{K}\mathbf{u}_\omega + \mathbf{R}\mathbf{z}_\omega \ge \mathbf{D}_\omega, \qquad (\boldsymbol{\nu}_\omega), \tag{54}$$

$$\mathbf{M}\mathbf{v}_\omega^r \le \mathbf{C}^r, \qquad (\boldsymbol{\lambda}_\omega), \tag{55}$$

$$\mathbf{H}\mathbf{v}_\omega^k \le \mathbf{C}^k \bar{\mathbf{r}}, \qquad (\boldsymbol{\rho}_\omega), \tag{56}$$

$$\mathbf{E}\mathbf{x}_\omega^r \le \mathbf{C}^r, \qquad (\boldsymbol{\delta}_\omega), \tag{57}$$

$$\mathbf{L}\mathbf{x}_\omega^k \le \mathbf{C}^k \bar{\mathbf{r}}, \qquad (\boldsymbol{\kappa}_\omega), \tag{58}$$

$$\mathbf{v}_\omega, \mathbf{u}_\omega, \mathbf{z}_\omega, \mathbf{x}_\omega \ge \mathbf{0}.$$

$Optimal\ solutions \rightarrow \boldsymbol{\pi}_\omega^{n+1}, \boldsymbol{\mu}_\omega^{n+1}, \boldsymbol{\nu}_\omega^{n+1}, \boldsymbol{\lambda}_\omega^{n+1}, \boldsymbol{\rho}_\omega^{n+1}, \boldsymbol{\delta}_\omega^{n+1}, \boldsymbol{\kappa}_\omega^{n+1}$

$\underset{(Z_{UB})}{Upper\ bound} \leftarrow \mathbf{f}^\intercal \bar{\mathbf{y}} + \mathbf{e}^\intercal \bar{\mathbf{g}} + \sum_{\omega \in \Omega} p_\omega Q(\bar{\mathbf{y}}, \bar{\mathbf{r}}, \omega)$

$if)\ \bar{\theta}_\omega < (\mathbf{a}_\omega^{n+1})^\intercal \bar{\mathbf{y}} + (\mathbf{c}_\omega^{n+1})^\intercal \bar{\mathbf{r}} + d_\omega^{n+1}$

$(\theta_\omega \ge (\mathbf{a}_\omega^{n+1})^\intercal \mathbf{y} + (\mathbf{c}_\omega^{n+1})^\intercal \mathbf{r} + d_\omega^{n+1}) \ \rightarrow\ Add\ to\ MP$

Acceleration method for Benders decomposition

See Appendix B.

Initialization problem (EVP)

$$\min \quad \mathbf{f}^\intercal \mathbf{y} + \mathbf{e}^\intercal \mathbf{g} + \mathbf{h}^\intercal \mathbf{v} + \mathbf{b}^\intercal \mathbf{u} + \boldsymbol{\beta}^\intercal \mathbf{z} + \mathbf{c}^\intercal \mathbf{x} \tag{59}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{g} \leq \mathbf{1}, \tag{60}$$

$$\mathbf{B}\mathbf{g} = \mathbf{r}, \tag{61}$$

$$\mathbf{G}\mathbf{r} \leq \mathbf{1}, \tag{62}$$

$$\mathbf{P}\mathbf{x} \leq \tilde{\mathbf{S}}\mathbf{y}, \tag{63}$$

$$\mathbf{U}\mathbf{u} + \mathbf{V}\mathbf{v} - \mathbf{T}\mathbf{x} = \mathbf{0}, \tag{64}$$

$$\mathbf{K}\mathbf{u} + \mathbf{R}\mathbf{z} \geq \tilde{\mathbf{D}}, \tag{65}$$

$$\mathbf{M}\mathbf{v}^r \leq \mathbf{C}^r, \tag{66}$$

$$\mathbf{H}\mathbf{v}^k \leq \mathbf{C}^k\mathbf{r}, \tag{67}$$

$$\mathbf{E}\mathbf{x}^r \leq \mathbf{C}^r, \tag{68}$$

$$\mathbf{L}\mathbf{x}^k \leq \mathbf{C}^k\mathbf{r}, \tag{69}$$

$$\mathbf{v}, \mathbf{u}, \mathbf{z}, \mathbf{x} \geq \mathbf{0}, \tag{70}$$

$$\mathbf{y} \in \{0,1\}^{|\mathcal{J}|}, \tag{71}$$

$$\mathbf{g}, \mathbf{r} \in \{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{72}$$

where

$$\tilde{\mathbf{S}} := \frac{1}{|\Omega|}\sum_{\omega\in\Omega} \mathbf{S}_\omega, \quad \tilde{\mathbf{D}} := \frac{1}{|\Omega|}\sum_{\omega\in\Omega} \mathbf{D}_\omega$$

(Typical method)

- Use any feasible solution of dual variables to generate initial optimality cut

(Acceleration method)

- **Step 1:** Solve **EVP** with Branch and Bound algorithm until the upper bound and lower bound gap is within 5%.
- **Step 2:** Get optimal solution $\bar{\mathbf{y}}, \bar{\mathbf{g}}, \bar{\mathbf{r}}$.
- **Step 3:** Solve **SUB(ω)** with obtained initialization solution $\bar{\mathbf{y}}, \bar{\mathbf{g}}, \bar{\mathbf{r}}$.
- **Step 4:** Get optimal dual solutions $\boldsymbol{\pi}_\omega, \boldsymbol{\nu}_\omega, \boldsymbol{\lambda}_\omega, \boldsymbol{\rho}_\omega, \boldsymbol{\kappa}_\omega, \boldsymbol{\delta}_\omega, \quad \forall\omega \in \Omega$.
- **Step 5:** Generate initial optimality cut with above optimal dual solutions.

IV. Computational Experiments

Description of the test instances

Test instances specifications (N= 40)

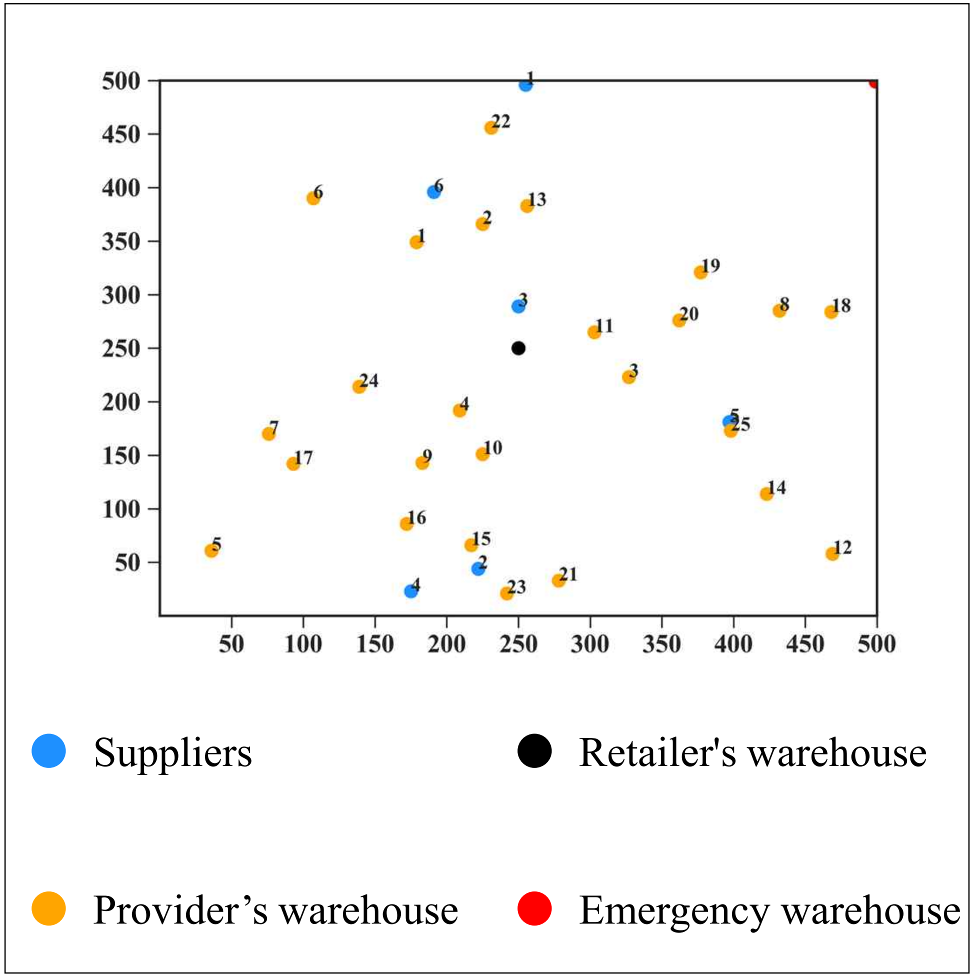
No.	XY	Total Vars	Binary Vars	Cont Vars	Cons	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{K}\rvert$	$\lvert\mathcal{M}\rvert$
1	100×100	29,103	153	28,950	14,000	2	3	10	5	3
2		32,899	131	32,768	15,104	2	3	8	8	2
3		73,844	324	73,520	26,000	3	4	10	8	4
4		88,204	304	87,900	30,000	3	4	10	10	3
5		88,803	483	88,320	34,800	3	3	12	10	4
6	300×300	146,859	471	146,388	51,168	4	3	12	13	3
7		149,404	544	148,860	48,480	3	4	12	15	3
8		289,355	680	288,675	76,200	4	5	15	15	3
9		434,705	1,355	433,350	109,080	5	5	18	15	5
10		467,405	1,205	466,200	112,200	5	5	15	20	4
11	500×500	532,806	906	531,900	114,600	5	6	15	20	3
12		561,605	1,805	559,800	135,360	5	5	18	20	5
13		1,046,606	2,506	1,044,100	210,800	6	6	20	25	5
14		1,049,606	4,006	1,045,600	213,800	6	6	20	25	8
15		1,738,566	5,766	1,732,800	334,560	7	6	24	30	8

Distributions for stochastic parameters value

D_{it}^{w}	S_{ijt}^{w}
$\mathcal{N}\left(\frac{179.06}{\lvert\mathcal{I}\rvert}, (\frac{91.18}{\lvert\mathcal{I}\rvert})^2\right)$	$\mathcal{N}\left(\frac{179.06}{\lvert\mathcal{I}\rvert}, (\frac{91.18}{\lvert\mathcal{I}\rvert})^2\right)$

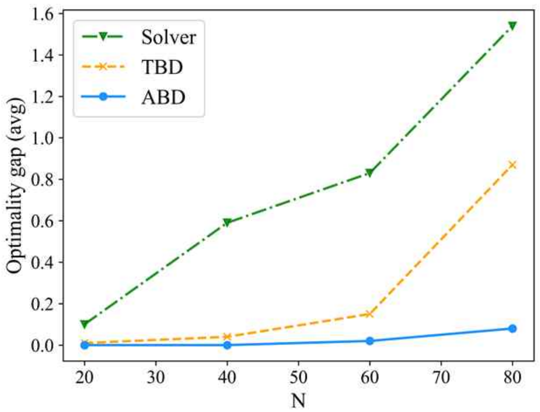
* Derived from the real-world E-commerce data

XY plane of test instance 14

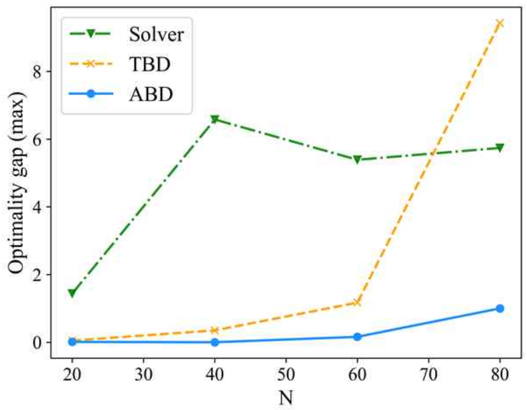


Performance of the Benders decomposition

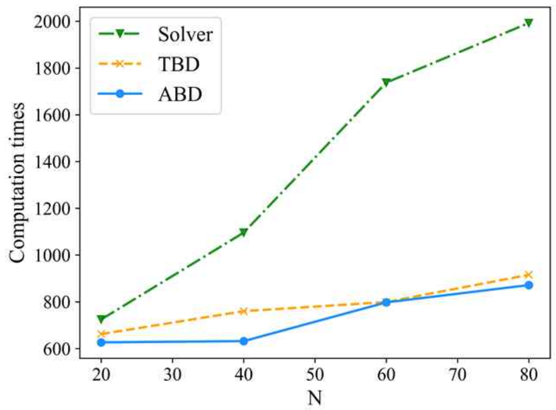
➤See Appendix C.



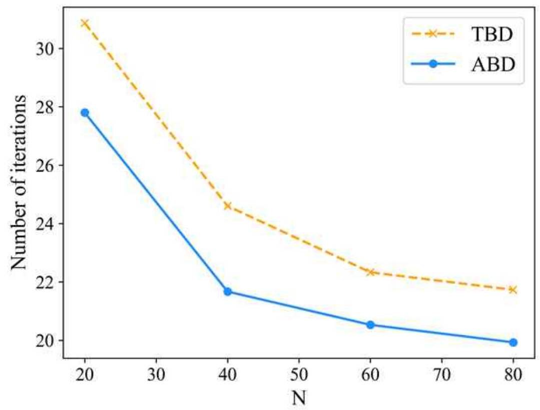
Gap (avg)



Gap (max)



CPUs



Itr

N	Solver			TBD				ABD			
	Gap (avg,max)	CPUs	# of t_{max}	Gap (avg,max)	CPUs	Itr	# of t_{max}	Gap (avg,max)	CPUs	Itr	# of t_{max}
20	(0.10, 1.44)	724.64	1	(0.01, 0.05)	661.91	30.87	1	(0.00, 0.01)	626.56	27.80	0
40	(0.59, 6.58)	1096.03	2	(0.04, 0.35)	0.04	24.60	2	(0.00, 0.00)	631.76	21.67	1
60	(0.83, 5.39)	1737.56	5	(0.15, 1.17)	0.15	22.33	2	(0.02, 0.16)	797.48	20.53	2
80	(1.54, 5.74)	1992.41	7	(0.87, 9.43)	0.87	21.73	2	(0.08, 1.00)	871.50	19.93	2

* $Gap := \frac{Best\ solution\ (Z_{UB}) - Best\ bound\ (Z_{LB})}{Best\ bound\ (Z_{LB})} \times 100(\%)$ * CPUs : Computation times (seconds) * Itr : Number of iterations

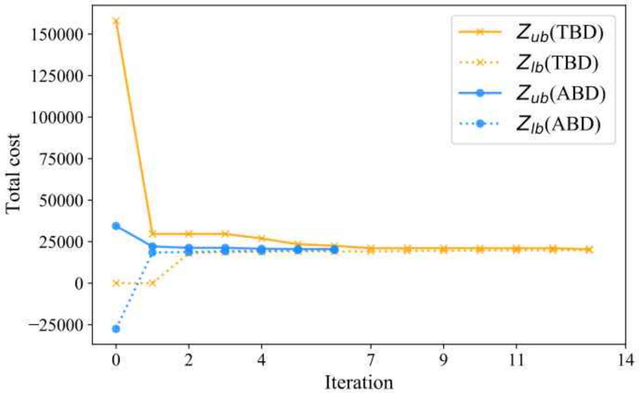
* # of t_{max} : Number of times experiments cannot be solved within time limit * Solver : Xpress-Optimizer

* TBD : Benders decomposition (Typical method) * ABD : Benders decomposition (Acceleration method)

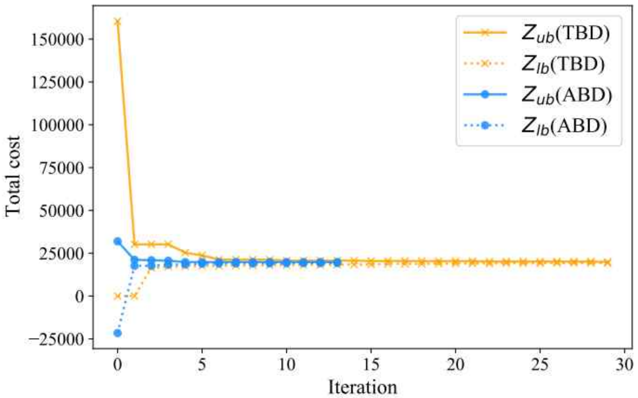
Performance of the Benders decomposition

Comparisons of the TBD and ABD

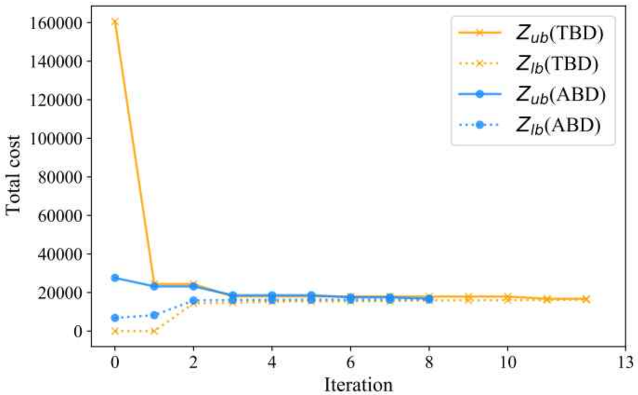
$\epsilon = 0.03$



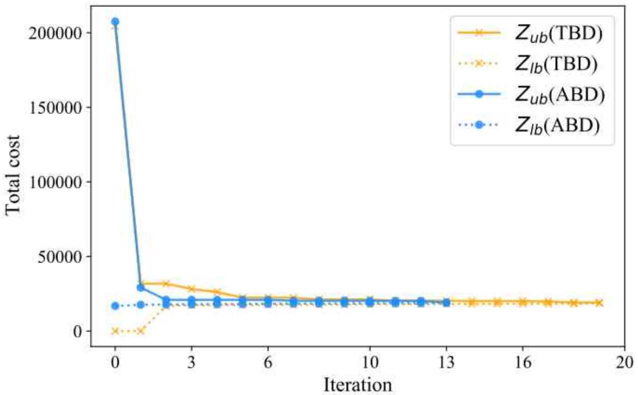
(a) Instance 12 ($N = 40$)



(b) Instance 13 ($N = 40$)



(c) Instance 14 ($N = 40$)



(d) Instance 15 ($N = 40$)

- ABD could generate a good initial cut in the primary stage
- ABD could converge faster than TBD

Quality of stochastic solution

Performance of sample average approximation

No.	N	LB	σ_{LB}	UB	σ_{UB}	EEV	σ_{EEV}	WS	VSS	EVPI
1	80	13,384.3	110.7	13,449.1	69.3	16,720.6	127.1	12,044.5	3,271.5	1,404.6
2	40	10,923.4	89.3	11,023.1	53.4	20,079.1	143.5	10,099.1	9,056.0	924.0
3	40	11,147.4	74.3	11,238.7	42.2	23,088.7	138.0	10,459.0	11,850.0	779.7
4	80	16,346.0	77.7	16,433.5	67.0	29,244.1	159.6	15,316.3	12,810.6	1,117.2
5	40	10,643.5	82.0	10,685.9	40.0	23,951.2	148.4	10,109.7	13,265.3	576.2
6	80	13,820.6	57.0	13,835.7	40.5	26,936.8	132.4	13,108.1	13,101.1	727.6
7	60	13,472.3	74.8	13,520.1	41.7	24,377.1	129.1	12,622.3	10,857.0	897.8
8	20	16,515.2	97.6	16,663.7	36.4	36,252.4	172.7	15,602.0	19,588.7	1,061.7
9	20	13,189.4	50.2	13,200.8	27.8	14,202.4	47.9	12,703.1	1,001.6	497.7
10	20	14,883.5	106.8	15,017.9	32.4	16,158.7	60.8	14,396.0	1,140.8	621.9
11	20	15,672.7	91.5	15,771.3	31.0	26,593.1	97.1	15,284.8	10,821.8	486.5
12	20	20,379.2	73.5	20,384.4	35.9	21,901.6	59.7	19,913.7	1,517.2	470.7

* EEV : expected result of using the EVS * WS : wait-and-see solution

* VSS : EEV – UB (value of the stochastic solution)

* EVPI : UB – WS (expected value of perfect information)

Optimality gap of stochastic solution and EVS

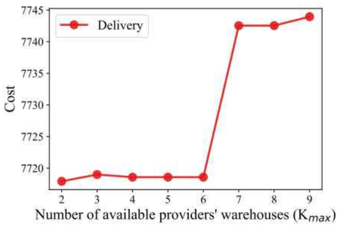
No.	Stochastic solution			EVS		
	Gap(abs)	Gap(%)	σ_{Gap}	Gap(abs)	Gap(%)	σ_{Gap}
1	64.8	0.48	130.60	3,336.3	24.93	168.51
2	99.7	0.91	104.08	9,155.7	83.82	169.06
3	91.3	0.82	85.46	11,941.3	107.12	156.71
4	87.5	0.54	102.61	12,898.1	78.91	177.50
5	42.4	0.40	91.22	13,307.7	125.03	169.52
6	15.1	0.11	69.88	13,116.2	94.90	144.10
7	47.7	0.35	85.69	10,904.8	80.94	149.20
8	148.5	0.90	104.14	19,737.2	119.51	198.34
9	11.4	0.09	57.40	1,013.0	7.68	69.41
10	134.4	0.90	111.56	1,275.2	8.57	122.87
11	98.7	0.63	96.56	10,920.4	69.68	133.41
12	5.2	0.03	81.81	1,522.4	7.47	94.69

* EVS : EVP solution

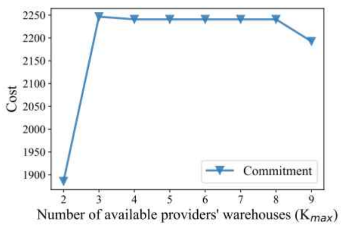
Effects of on-demand warehousing system on supply chain

Impact of different number of available providers' warehouses on cost

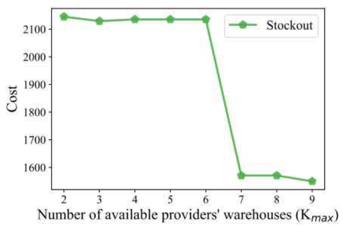
K_{max}	Utilization	Cost						
		Total	Delivery	Commitment	Stockout	Suppliers	Transportation	Inventory
1	15	32,406.9	6,534.7	1,120.4	19,985.5	1,134.8	3,566.1	65.4
2	25	16,360.0	7,717.9	1,885.6	2,145.3	1,134.8	3,397.2	79.2
3	30	15,532.5	7,719.0	2,246.5	2,129.3	1,134.8	2,229.4	73.4
4	30	15,259.9	7,718.6	2,240.8	2,135.5	1,134.8	1,969.3	61.0
5	30	15,259.9	7,718.6	2,240.8	2,135.5	1,134.8	1,969.3	61.0
6	30	15,259.9	7,718.6	2,240.8	2,135.5	1,134.8	1,969.3	61.0
7	30	15,098.2	7,742.5	2,240.8	1,570.6	1,810.1	1,668.4	65.7
8	30	15,098.2	7,742.5	2,240.8	1,570.6	1,810.1	1,668.4	65.7
9	29	14,986.9	7,743.9	2,192.4	1,549.6	1,810.1	1,634.4	56.5
15	29	14,986.9	7,743.9	2,192.4	1,549.6	1,810.1	1,634.4	56.5
20	29	14,986.9	7,743.9	2,192.4	1,549.6	1,810.1	1,634.4	56.5



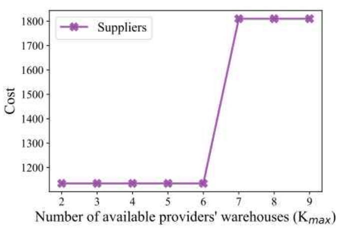
(a) Delivery service cost



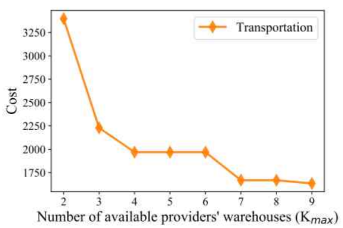
(b) Commitment cost



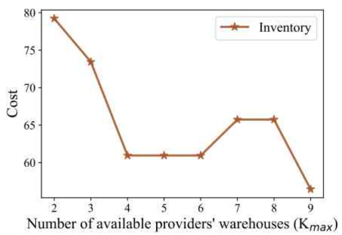
(c) Stockout cost



(d) Suppliers investment cost



(e) Transportation cost



(f) Inventory holding cost

- Number of available providers' warehouses ↑

  ⇒ Total cost ↓

  ⇒ Stockout cost ↓

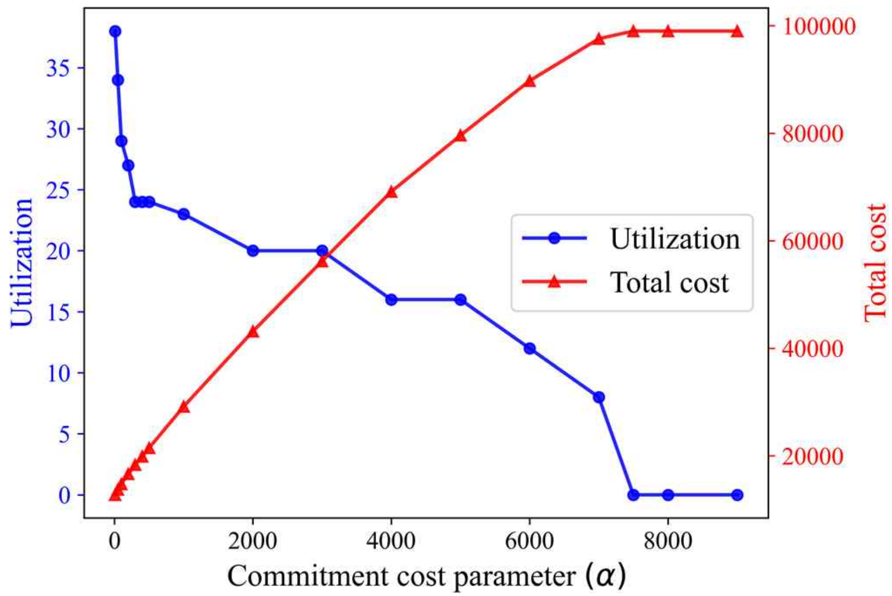
  ⇒ Transportation cost ↓

  ⇒ Delivery cost ↑

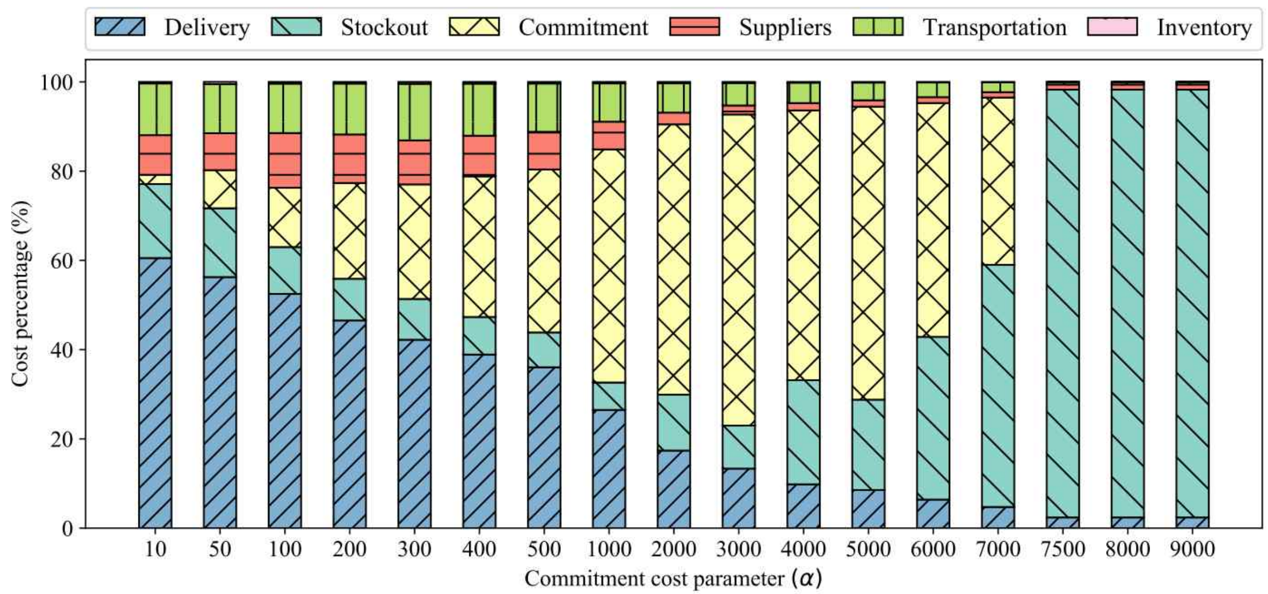
* Utilization $:= \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} r^k_{mt}$
* Delivery service cost $:= \frac{1}{N}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega} b_i\left(u^{r\omega}_{it} + u^{e\omega}_{it} + \sum_{k\in\mathcal{K}} u^{k\omega}_{it}\right)$
* Commitment cost $:= \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} m\alpha\gamma^m g^k_{mt}$
* Stockout cost $:= \frac{1}{N}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega} \beta_i z^\omega_{it}$
* Suppliers investment cost $:= \sum_{j\in\mathcal{J}} F_j y_j$
* Transportation cost $:= \frac{1}{N}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega}\left(c^r_{ij}x^{r\omega}_{ijt} + c^e_{ij}x^{e\omega}_{ijt} + \sum_{k\in\mathcal{K}} c^k_{ij}x^{k\omega}_{ijt}\right)$
* Inventory holding cost $:= \frac{1}{N}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega}\left(h^r_i v^{r\omega}_{it} + h^e_i v^{e\omega}_{it} + \sum_{k\in\mathcal{K}} h^k_i v^{k\omega}_{it}\right)$

Sensitivity analysis on the commitment cost parameter

Utilization and total cost varying the commitment cost

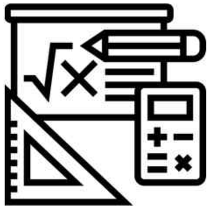


Share of the total cost for each type of cost varying the commitment cost

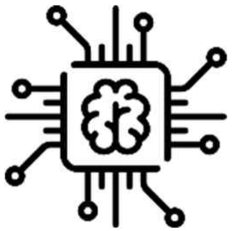


V. Conclusions

Contributions



Two-stage stochastic programming model → On-demand warehousing system



Benders decomposition (acceleration method) + Sample average approximation

Evaluation of performances → Benders decomposition & MIP solver

Further studies

Real-world data	⇨	Test problem instance
Valid inequalities & Pareto optimal cut	⇨	Reduce computation time
Accommodate other types of uncertainties	⇨	Capacity of warehouses

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Appendix A. Sample average approximation

Algorithm 1 Sample Average Approximation

Initializations:
  $N \leftarrow 0$;
  $Gap^{rel}_{MNN'} \leftarrow \infty$;
while $Gap^{rel}_{MNN'} > \epsilon_{saa}$ or $Gap^{rel}_{MNN'} < 0$ **do**
  $N \leftarrow N + 20$;
  Estimation of lower bound:
    Generate M independent samples of size N scenarios, $(\omega_1^m, \cdots, \omega_N^m)$, $\forall m \in \{1, \cdots, M\}$;
    for each $m \in \{1, \cdots, M\}$ **do**
      $\hat{\psi}_N^m \leftarrow$ optimal objective value of the corresponding SAA problem;
      $\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m, \hat{\mathbf{r}}_N^m \leftarrow$ optimal solution of the corresponding SAA problem;
    end for
    $\bar{\psi}_{MN} \leftarrow \frac{1}{M}\sum_{m=1}^{M} \hat{\psi}_N^m$;
    $\sigma^2_{\bar{\psi}_{MN}} \leftarrow \frac{1}{M(M-1)}\sum_{m=1}^{M}\left(\hat{\psi}_N^m - \bar{\psi}_{MN}\right)^2$;
  Estimation of upper bound:
    Generate a samples of size N' scenarios, $(\omega_1, \cdots, \omega_{N'})$, $N \ll N'$;
    for each $m \in \{1, \cdots, M\}$ **do**
      $\bar{f}_{N'}(\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m, \hat{\mathbf{r}}_N^m) \leftarrow \mathbf{f}^\intercal \hat{\mathbf{y}}_N^m + \mathbf{e}^\intercal \hat{\mathbf{g}}_N^m + \frac{1}{N'}\sum_{n=1}^{N'} Q(\hat{\mathbf{y}}_N^m, \hat{\mathbf{r}}_N^m, \omega_n)$;
    end for
    $m^* \leftarrow \text{argmin}_m \ \bar{f}_{N'}(\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m, \hat{\mathbf{r}}_N^m)$, $\forall m \in \{1, \cdots, M\}$;
    $\hat{\mathbf{y}} \leftarrow \hat{\mathbf{y}}_N^{m^*}$, $\hat{\mathbf{g}} \leftarrow \hat{\mathbf{g}}_N^{m^*}$, $\hat{\mathbf{r}} \leftarrow \hat{\mathbf{r}}_N^{m^*}$;
    $\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) \leftarrow \mathbf{f}^\intercal \hat{\mathbf{y}} + \mathbf{e}^\intercal \hat{\mathbf{g}} + \frac{1}{N'}\sum_{n=1}^{N'} Q(\hat{\mathbf{y}}, \hat{\mathbf{r}}, \omega_n)$;
    $\sigma^2_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) \leftarrow \frac{1}{N'(N'-1)}\sum_{n=1}^{N'}\left(\mathbf{f}^\intercal \hat{\mathbf{y}} + \mathbf{e}^\intercal \hat{\mathbf{g}} + Q(\hat{\mathbf{y}}, \hat{\mathbf{r}}, \omega_n) - \bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})\right)^2$;
  Estimation of SAA gap:
    $Gap_{MNN'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) \leftarrow \hat{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) - \hat{\psi}_{MN}$;
    $\sigma^2_{Gap_{MNN'}} \leftarrow \sigma^2_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) + \sigma^2_{\bar{\psi}_{MN}}$;
    $Gap^{rel}_{MNN'} \leftarrow \frac{(\hat{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) - \bar{\psi}_{MN})}{\bar{\psi}_{MN}} \times 100(\%)$
end while
Return: $N, \bar{\psi}_{MN}, \sigma^2_{\bar{\psi}_{MN}}, \bar{f}_{N'}, \sigma^2_{N'}, Gap_{MNN'}, \sigma^2_{Gap_{MNN'}}, Gap^{rel}_{MNN'}$

Appendix B. Benders decomposition with acceleration

Algorithm 2 Benders Decomposition Algorithm (Acceleration Method)

Initializations
  $Z_{ub} \leftarrow \infty,\ Z_{lb} \leftarrow -\infty, itr \leftarrow 1;$
  solve EVP and get $\bar{\mathbf{y}}, \bar{\mathbf{g}}, \bar{\mathbf{r}};$
for each $\omega \in \Omega$ **do**
  solve $SUB(\omega)$ and get dual solution;
  get $(\mathbf{a}_{\omega}^{itr})^{\intercal}, (\mathbf{c}_{\omega}^{itr})^{\intercal}, d_{\omega}$ with dual solution;
  add an initial optimality cut to MP;
end for
while $Z_{ub} - Z_{lb} \geq \epsilon \times Z_{lb}$ **do**
  solve MP and get $\bar{\mathbf{y}}, \bar{\mathbf{g}}, \bar{\mathbf{r}}, \bar{\theta}_{\omega}, \forall \omega \in \Omega;$
  $Z_{lb} \leftarrow \max \left\{ Z_{lb}, \mathbf{f}^{\intercal}\bar{\mathbf{y}} + \mathbf{e}^{\intercal}\bar{\mathbf{g}} + \sum_{\omega \in \Omega} p_{\omega}\bar{\theta}_{\omega} \right\};$
  for each $\omega \in \Omega$ **do**
    solve $SUB(\omega)$ and get dual solution;
    get $(\mathbf{a}_{\omega}^{itr+1})^{\intercal}, (\mathbf{c}_{\omega}^{itr+1})^{\intercal}, d_{\omega}$ with dual solution;
    store the optimal objective value $Q(\bar{\mathbf{y}}, \bar{\mathbf{r}}, \omega);$
    if $\bar{\theta}_{\omega} < Q(\bar{\mathbf{y}}, \bar{\mathbf{r}}, \omega)$ **then**
      add an optimality cut to MP;
    end if
  end for
  if $Z_{ub} > \mathbf{f}^{\intercal}\bar{\mathbf{y}} + \mathbf{e}^{\intercal}\bar{\mathbf{g}} + \sum_{\omega \in \Omega} p_{\omega} Q(\bar{\mathbf{y}}, \bar{\mathbf{r}}, \omega)$ **then**
    $Z_{ub} \leftarrow \mathbf{f}^{\intercal}\bar{\mathbf{y}} + \mathbf{e}^{\intercal}\bar{\mathbf{g}} + \sum_{\omega \in \Omega} p_{\omega} Q(\bar{\mathbf{y}}, \bar{\mathbf{r}}, \omega);$
    $\mathbf{y}^* \leftarrow \bar{\mathbf{y}}, \mathbf{g}^* \leftarrow \bar{\mathbf{g}}, \mathbf{r}^* \leftarrow \bar{\mathbf{r}};$
    $\mathbf{u}_{\omega}^* \leftarrow \bar{\mathbf{u}}_{\omega}, \mathbf{v}_{\omega}^* \leftarrow \bar{\mathbf{v}}_{\omega}, \mathbf{x}_{\omega}^* \leftarrow \bar{\mathbf{x}}_{\omega}, \forall \omega \in \Omega;$
  end if
  $itr \leftarrow itr + 1$
end while
Return: $Z_{ub}, Z_{lb}, \mathbf{y}^*, \mathbf{g}^*, \mathbf{r}^*, \mathbf{u}_{\omega}^*, \mathbf{x}_{\omega}^*, \mathbf{v}_{\omega}^*, \forall \omega \in \Omega;$

SCM Lab.

Appendix C. Computation results of the proposed algorithm

No.	Method	N											
		20			40			60			80		
		Gap	CPUs	Itr	Gap	CPUs	Itr	Gap	CPUs	Itr	Gap	CPUs	Itr
1	Solver	0.01	9.10	-	0.00	20.88	-	0.00	31.83	-	0.00	44.76	-
	TBD	0.00	1.52	10	0.00	4.15	12	0.00	4.76	12	0.00	8.66	13
	ABD	0.00	1.43	8	0.00	3.69	10	0.01	3.95	9	0.00	5.86	9
2	Solver	0.00	17.02	-	0.00	12.01	-	0.00	26.46	-	0.00	32.91	-
	TBD	0.01	11.39	16	0.00	3.48	12	0.00	5.76	14	0.00	9.59	14
	ABD	0.01	15.78	16	0.00	3.75	12	0.00	3.52	9	0.00	10.86	13
3	Solver	0.00	25.28	-	0.00	85.65	-	0.00	106.62	-	0.00	162.18	-
	TBD	0.00	5.38	10	0.00	21.51	14	0.00	22.90	14	0.00	42.72	14
	ABD	0.00	5.87	9	0.00	20.17	11	0.00	29.33	15	0.00	38.06	12
4	Solver	0.00	37.27	-	0.00	41.29	-	0.00	78.91	-	0.00	137.13	-
	TBD	0.00	21.34	24	0.00	28.35	20	0.00	20.28	13	0.00	59.74	21
	ABD	0.00	25.21	23	0.00	25.65	17	0.00	31.02	14	0.00	46.51	17
5	Solver	0.00	261.04	-	0.00	81.00	-	0.00	220.67	-	0.00	215.84	-
	TBD	0.00	59.89	26	0.00	17.97	13	0.00	64.15	19	0.00	75.06	17
	ABD	0.00	86.06	25	0.00	24.51	13	0.01	43.07	13	0.00	46.71	11
6	Solver	0.00	34.65	-	0.00	99.10	-	0.00	156.24	-	0.00	186.76	-
	TBD	0.01	25.14	20	0.00	28.85	14	0.00	38.02	13	0.00	50.92	14
	ABD	0.01	20.99	17	0.00	24.67	11	0.00	30.42	10	0.00	27.51	8
7	Solver	0.00	95.04	-	0.00	156.42	-	0.00	270.24	-	0.00	406.06	-
	TBD	0.00	79.92	28	0.00	37.88	17	0.01	137.00	27	0.01	167.79	26
	ABD	0.00	79.99	28	0.00	34.59	13	0.01	149.66	27	0.00	180.35	25
8	Solver	0.00	644.87	-	0.01	1,488.56	-	2.72	3,600*	-	3.94	3,600*	-
	TBD	0.01	1,812.62	42	0.01	849.53	50	0.01	352.67	26	0.00	961.77	31
	ABD	0.01	1,855.33	38	0.01	865.72	49	0.00	269.13	19	0.00	975.94	31
9	Solver	0.00	945.19	-	0.00	1,377.55	-	0.00	2,792.00	-	1.99	3,600*	-
	TBD	0.00	875.93	43	0.00	463.74	23	0.00	519.25	24	0.00	842.90	27
	ABD	0.00	1,176.35	43	0.00	240.60	18	0.00	497.58	21	0.00	772.08	23
10	Solver	0.01	313.20	-	0.00	858.58	-	1.74	3,600*	-	3.12	3,600*	-
	TBD	0.00	170.04	31	0.00	246.00	25	0.00	762.02	32	0.01	540.72	22
	ABD	0.00	176.85	28	0.00	201.56	20	0.00	491.67	26	0.00	453.25	19
11	Solver	0.01	285.13	-	0.00	766.73	-	0.00	1752.75	-	0.00	3,500.58	-
	TBD	0.00	116.37	28	0.01	207.00	25	0.00	282.96	22	0.00	448.74	26
	ABD	0.01	67.85	18	0.00	187.64	22	0.00	360.37	24	0.00	466.15	24
12	Solver	0.00	455.63	-	0.00	1,983.03	-	0.00	2,627.69	-	0.64	3,600*	-
	TBD	0.01	504.14	37	0.00	469.77	28	0.00	1,095.53	28	0.00	970.46	26
	ABD	0.00	471.05	32	0.00	296.14	21	0.00	899.16	24	0.00	829.04	23
13	Solver	1.44	3,600*	-	2.28	3,600*	-	5.39	3,600*	-	5.74	3,600*	-
	TBD	0.05	3,600*	76	0.30	3,600*	52	0.99	3,600*	43	3.64	3,600*	33
	ABD	0.01	3,156.88	70	0.03	3,600*	51	0.12	3,600*	44	0.21	3,600*	37
14	Solver	0.00	1,217.37	-	0.00	2,269.65	-	0.24	3,600*	-	2.82	3,600*	-
	TBD	0.00	509.10	25	0.00	1,821.07	27	0.00	1478.69	20	0.00	2,352.70	23
	ABD	0.01	371.06	17	0.01	1,410.55	21	0.00	1,953.34	20	0.01	2,020.25	20
15	Solver	0.00	2,928.81	-	6.58	3,600*	-	2.37	3,600*	-	4.84	3,600*	-
	TBD	0.00	2,135.87	47	0.35	3,600*	37	1.17	3,600*	28	9.43	3,600*	19
	ABD	0.00	1,887.76	45	0.00	2,537.23	36	0.16	3,600*	33	1.00	3,600*	27

7	Solver	0.00	95.04	-	0.00	156.42	-	0.00	270.24	-	0.00	406.06	-
	TBD	0.00	79.92	28	0.00	37.88	17	0.01	137.00	27	0.01	167.79	26
	ABD	0.00	79.99	28	0.00	34.59	13	0.01	149.66	27	0.00	180.35	25
8	Solver	0.00	644.87	-	0.01	1,488.56	-	2.72	3,600*	-	3.94	3,600*	-
	TBD	0.01	1,812.62	42	0.01	849.53	50	0.01	352.67	26	0.00	961.77	31
	ABD	0.01	1,855.33	38	0.01	865.72	49	0.00	269.13	19	0.00	975.94	31

* Time limit was reached

Thank you

Q&A

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