A Distributed Inventory Coordination for Single Warehouse Multiple Retailer Problems

DongJoo Lee[1] & In-Jae Jeong[2]

Department of Industrial Engineering, Hanyang University
17 Haengdang-dong, Seongdong-gu, Seoul 133-791, Korea
[1]E-mail: fineldj@hanyang.ac.kr
[2]E-mail: ijeong@hanyang.ac.kr

In this paper, we consider a distributed inventory coordination for single warehouse and multiple retailer problems. The objective is to minimize the sum of the setup cost and inventory holding cost incurring among warehouse and retailers in a distributed manner. We apply the alternating direction method which utilized Augmented Lagrangian function and multiplier updating scheme to solve the problem. The procedure solves the problem by communicating partial information between a coordinator, the warehouse and retailers. A numerical example shows that the algorithm converges to an optimal solution within a reasonable time.

Keywords: SCM, Distributed inventory coordination, Alternating direction method

1. Introduction

Most manufacturing enterprises are organized into networks of manufacturing and distribution sites that procure raw material, process them into finished goods, and distribute the finish goods to customers. Inventories exist throughout the supply chain in various forms for various reasons and at any manufacturing point, they may exist as raw materials, work in progress, or finished goods [3].

We consider a single warehouse and multiple retailer (SWMR) in supply chain management. We assume only limited information sharing is possible between warehouse and retailers in a distributed environment. The objective is to minimize the sum of the setup cost and inventory holding cost incurring in warehouse and retailers.

It is well known that the optimal policies for SWMR is very complex (Graves and Schwarz 1977). Most researchers have restricted their attention to stationary policies and nested policies (Schwarz 1973, Schwarz and Schrage 1975, Graves and Schwarz 1977, and Maxwell and Muckstadt 1985). Lu and Posner (1994) reconsider this problem by restricting to integer-ratio policies. Abdul-Jalbar (2006) proposed a heuristic procedure to compute efficient single-cycle polices.

All the work mentioned above are centralized algorithms which assume that all the problem data are given to a single decision maker.

In this paper, we consider a distributed environment of SWMR problem where the warehouse and retailers can share only partial information of its own system. We assume a fixed base planning period, and that all the order intervals are power-of-two multiples of periods.

The remainder of the paper is organized as follow. In Section 2, we describe a SWMR problem and in Section 3, we propose overall implementation procedure and algorithm. In Section 4, a numerical example is presented.

2. Problem Description

We assume that base period is fixed and restrict to power-of-two policies as follows:

- Each retailer face a constant demand.
- No shortage or backlogging.
- Replenishment is instantaneous.

Costs at warehouse and each retailer consist of a setup cost per order and a holding cost. If $h_i^{'}$ is holding cost at retailer i and $h_0^{'} \le h_i^{'}$ is a holding cost at warehouse, then echelon holding cost is defined as $h_0 = h_0^{'}$ and $h_i = h_i^{'} - h_0^{'}$.

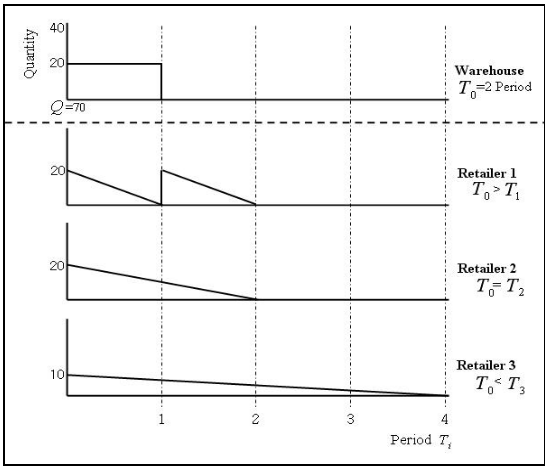

Figure 1. Single warehouse and multiple retailers

This model consists of single warehouse that serves as sole supplier to all multiple retailers (Figure 1). Each retailer sells only single type of item and all demands must be satisfied without shortage or backlogging. Each retailer faces a constant demand that is met by placing orders to the warehouse. And then, the warehouse orders from an outside supplier and replenishes the retailers which in turn satisfy customer demand. The total cost is the sum of setup cost and inventory holding cost in warehouse and each retailer. A SWMR problem can be described using the following notations and variables [1][5]**:**

Notations

D_i : Constant demand rate at retailer i , $i = 1, 2, ..., n$.

K_i : Setup cost at retailer i , $i = 1, 2, ..., n$.

K_0 : Setup cost at warehouse.

$h_i^{'}$: Unit holding cost at retailer i , $i = 1, 2, ..., n$.

$h_0^{'}$: Unit holding cost at warehouse.

T_B : Base planning period.

Decision variables

T_0 : Order period of warehouse

T_i : Order period of retailer i , $i = 1, 2, ..., n$

Let for simplicity, $g_i = \frac{1}{2} h_i D_i$ *and* $g^i = \frac{1}{2} h_0 D_i$

$$Min \;\; \frac{K_0}{T_0} + \sum_{i=1}^{n} \left(\frac{K_i}{T_i} + g_i T_i + g^i \max(T_0, T_i) \right) \quad (1)$$

$$s.t. \;\; T_i = 2^k T_B, \quad i = 0, 1, \ldots, n \qquad 0 \le k, \quad \text{integer} \quad (2)$$

(1) represents the objective function of SWMR problem which is the sum of setup cost and inventory holding costs. (2) is the power-of-two constraint. Roundy (1985) proposed a 94% effective algorithm to solve the problem which is a centralized algorithm. However in this paper, we

propose a decentralized algorithm which is also 94% effective.

3. Limited information with SWMR

We examine the SWMR problem in distributed environment.

3.1 Alternating direction method

This method operates in cycles, where in each cycle we minimize the Augmented Lagrangian function with respect to one set of variables, then minimize it with respect to the remaining variables, and then carry out a multiplier update [2]. This method guarantees optimal convergence under a special condition about objective function and decision variables (see Dimitri and John [2] for derails).

Let $\max(T_{0,}T_i) = u_i$ then the following conditions must be satisfied

$$\begin{aligned} &T_0 \le u_i, \quad u_i = T_0 + \alpha_i \\ &T_i \le u_i, \quad u_i = T_i + \beta_i \end{aligned} \tag{3}$$

We consider the application of alternating direction method for SWMR. SWMR problem without power-of-two policy restriction can be transformed as follows:

The total average cost is

$$Min \quad \frac{K_0}{T_0} + \sum_{i=1}^{n} \left(\frac{K_i}{T_i} + g_i T_i + g^i u_i \right) \tag{4}$$

$$\begin{aligned} s.t. \quad & u_i = T_0 + \alpha_i \\ & u_i = T_i + \beta_i \end{aligned} \tag{5}$$

$$\alpha_i, \beta_i \le u_i, \ \ 0 \le \alpha_i, \beta_i, u_i$$

Since the objective function and constraints are convex, we can apply Alternating Direction Method by constructing Lagrangian function as follows:

$$\begin{aligned} &L_c(T_0, T_i, u_i, \alpha_i, \beta_i, p_i, q_i) \\ &= \frac{K_0}{T_0} + \sum_{i=1}^{n} \left(\frac{K_i}{T_i} + g_i T_i + g^i u_i \right) \\ &\quad + \sum_{i=1}^{n} p_i (u_i - T_0 - \alpha_i) + \sum_{i=1}^{n} q_i (u_i - T_i - \beta_i) \\ &\quad + \frac{c_1}{2} \sum_{i=1}^{n} (u_i - T_0 - \alpha_i)^2 + \frac{c_2}{2} \sum_{i=1}^{n} (u_i - T_i - \beta_i)^2 \end{aligned} \tag{6}$$

The alternating direction method of multipliers is given by

$$\begin{aligned} \frac{\partial L}{\partial T_{0(t+1)}} &= \frac{-K_0}{T_0{}^2{}_{(t+1)}} - \sum_{i=1}^{n} p_{i(t)} \\ &\quad - c_1 \sum_{i=1}^{n} \left(u_{i(t)} - T_{0(t+1)} - \alpha_{i(t)} \right) = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{\partial L}{\partial T_{i(t+1)}} &= \frac{-K_i}{T_i{}^2{}_{(t+1)}} + g_i T_{i(t+1)} \\ &\quad - q_{i(t)} - c_2 \left(u_{i(t)} - T_{i(t+1)} - \beta_{i(t)} \right) = 0 \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{\partial L}{\partial u_{i(t+1)}} &= g^i + p_{i(t)} + q_{i(t)} \\ &\quad + c_1 \left(u_{i(t+1)} - T_{0(t+1)} - \alpha_{i(t)} \right) \\ &\quad + c_2 \left(u_{i(t+1)} - T_{i(t+1)} - \beta_{i(t)} \right) = 0 \end{aligned} \tag{9}$$

$$\frac{\partial L}{\partial \alpha_{i(t+1)}} = -p_{i(t)} - c_1 (u_{i(t)} - T_{0(t+1)} - a_{i(t+1)}) = 0 \tag{10}$$

$$\frac{\partial L}{\partial \beta_{i(t+1)}} = -q_{i(t)} - c_2 \left(u_{i(t)} - T_{i(t+1)} - \beta_{i(t+1)} \right) = 0 \tag{11}$$

$$p_{i(t+1)} = p_{i(t)} + c_3 \left(u_{i(t+1)} - T_{0(t+1)} - \alpha_{i(t+1)} \right) \tag{12}$$

$$q_{i(t+1)} = q_{i(t)} + c_4 \left(u_{i(t+1)} - T_{i(t+1)} - \beta_{i(t+1)} \right) \tag{13}$$

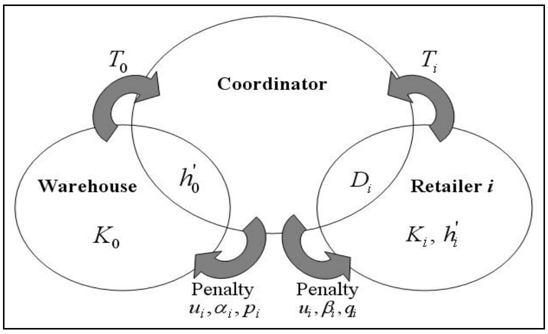

Figure 2. Information flow of algorithm

The proposed method operates in cycles, where we minimize the Augmented Lagrangian with respect to period T_0 of warehouse in each cycle, then minimize it with respect to period T_i remaining retailers, and then updates multiplier (penalty) $u_i, \alpha_i, \beta_i, p_i, q_i$ by coordinator. The algorithm stops if a certain stopping criteria is satisfied (Figure 2).

Once the optimal solution of (4) and (5) is found, we apply power-of-two policy for the optimal solution.

3.2 Proposed algorithm

Step1 Initialization: $t=1$, Initialize arbitrary.

$T_{0(t)}, T_{i(t)}, u_{i(t)}, \alpha_{i(t)}, \beta_{i(t)}, p_{i(t)}, q_{i(t)}$

Step2 Warehouse: Update $T_{0(t+1)}$ by equation (7).

Step3 Retailers: Update $T_{i(t+1)}$ by equation (8).

Step4 Coordinator: Update $u_{i(t+1)}, \alpha_{i(t+1)}, \beta_{i(t+1)}, p_{i(t+1)}, q_{i(t+1)}$ by equation (9)~(13).

Step5 Stop criterion:

If $\left|T_{0(t+1)} - T_{0(t)}\right|$ *and* $\left|T_{i(t+1)} - T_{i(t)}\right| < \varepsilon$ Stop.

Otherwise $t = t+1$ go to *Step2*.

Step6: Apply power-of-two policy for the solution found.

4. Example

We consider a simple example of SWMR problem in Table 1.

Table 1. Parameter for a sample SWMR problem

	W	R_1	R_2	R_3	R_4
K_i	400	100	200	50	120
D_i	-	1000	3000	10000	12000
h_i'	1	2	2	2	2

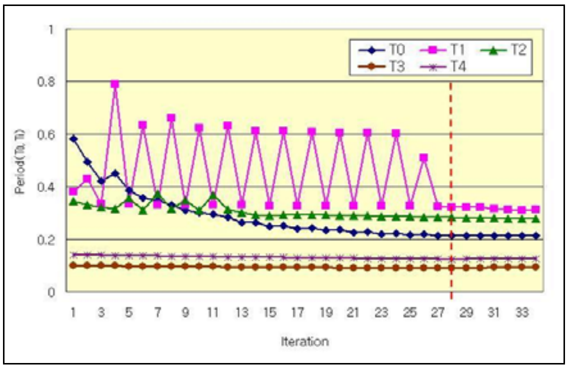

Figure 3. Performance of alternating direction method for the example

Figure 3 shows that the proposed algorithm converges to a solution after 28 iterations. Since Alternating Direction Method guarantees optimal convergence, the solution found by the proposed algorithm is also optimal solution.

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